

# Interplay between Quantum and Coherent Effects in Optics

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## Abstract

A technique is presented for treating strongly nonstationary and transient processes in optics, permitting one to take into account both types of competing with each other effects, quantum as well as coherent. The main equations for describing the interplay between these two kinds of effects are derived. The possibility of influencing coherent optical phenomena by preparing special quantum states of matter is discussed.

**Keywords:** coherent optical phenomena, quantum effects, squeezed vacuum.

## 1. Introduction

Coherent and quantum phenomena are often treated as antagonists, since the former are, to some extent, close to classical ones<sup>1</sup>. When describing one of these phenomena, one usually employs rather different approximations. For example, a very common way of considering coherent phenomena is by invoking the semiclassical approximation, which makes it possible to give a transparent picture of such phenomena. But the semiclassical approximation kills all quantum correlations, eliminating by this all quantum effects. Such a neglect of the latter may be admissible if the studied coherent process is stationary or a strong coherence is imposed on the system by an external field. However, if one investigates a self-organized growth of coherence and when the coherent phenomena are transient or intermittent, then quantum effects may essentially influence the features of coherent phenomena. In that case, it is necessary to take account of both types of effects. For this purpose, one may resort to the consideration of higher-order correlation functions. But then, the evolution equations become so much complicated that essentially nonstationary states are hardly treatable.

In this report, a method is presented, which combines the transparency of the semiclassical approximation with the possibility of taking account of quantum effects. The idea of the

method reminds, to some extent, quantization of soliton solutions in quantum field theory<sup>2</sup>. But the most important concept of the method is the notice that quantum and coherent effects occur on rather different scales. Quantum effects are usually connected with short-range fast fluctuations, while coherent phenomena are mainly long-range in space and slow in time, as compared to the quantum ones. The occurrence of different spatio-temporal scales allows the development of *Scale Separation Approach*<sup>3–7</sup>. The general idea of separating different scales is, of course, known. However, its realization, as applied to optics in the report below is new. The pivotal novel technique is *Quantization of Local Fluctuations*.

## 2. Resonant Atoms in Matter

The system of resonant atoms inside matter is described by the Hamiltonian

$$\hat{H} = \hat{H}_a + \hat{H}_f + \hat{H}_m + \hat{H}_{af} + \hat{H}_{mf} . \quad (1)$$

Here, the first term

$$\hat{H}_a = \sum_{i=1}^N \omega_0 \left( S_i^z + \frac{1}{2} \right) \quad (2)$$

is the Hamiltonian of resonant atoms, with transition frequency  $\omega_0$ , where  $S_i^\alpha$  are pseudospin operators. The field Hamiltonian is

$$\hat{H}_f = \frac{1}{8\pi} \int (\mathbf{E}^2 + \mathbf{B}^2) d\mathbf{r} . \quad (3)$$

The Gaussian system of units is used, with setting  $\hbar \equiv 1$ . Electric and magnetic fields are expressed through the vector potential,

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} , \quad \mathbf{B} = \nabla \times \mathbf{A} .$$

The Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$  will be employed. This calibration is convenient for eliminating field variables in the following evolution operations. The Hamiltonian  $\hat{H}_m$  models the material incorporating the atoms. The atom-field interaction is presented by the dipole Hamiltonian

$$\hat{H}_{af} = - \sum_{n=1}^N \left( \frac{1}{c} \mathbf{J}_i \cdot \mathbf{A}_i + \mathbf{P}_i \cdot \mathbf{E}_{0i} \right) , \quad (4)$$

in which  $\mathbf{A}_i \equiv \mathbf{A}(\mathbf{r}_i, t)$  is the vector-potential operator,  $\mathbf{E}_{0i} \equiv \mathbf{E}_0(\mathbf{r}_i, t)$  is a classical external field. The transition current and transition dipole operators are

$$\mathbf{J}_i = i\omega_0 (\mathbf{d}S_i^+ - \mathbf{d}^*S_i^-) , \quad \mathbf{P}_i = \mathbf{d}S_i^+ + \mathbf{d}^*S_i^- , \quad (5)$$

where  $\mathbf{d}$  is a transition dipole and  $S_i^\pm \equiv S_i^x \pm iS_i^y$ . Finally, the matter-field interaction is described by the Hamiltonian

$$\hat{H}_{mf} = - \frac{1}{c} \int \mathbf{j}_m(\mathbf{r}, t) \cdot \mathbf{A}(\mathbf{r}, t) d\mathbf{r} , \quad (6)$$

in which  $\mathbf{j}_m(\mathbf{r}, t)$  is a local density of current in matter.

The Heisenberg equations of motion for the pseudospin operators yield the equation

$$\frac{dS_i^-}{dt} = -i\omega_0 S_i^- + 2(k_0 \mathbf{d} \cdot \mathbf{A}_i - i\mathbf{d} \cdot \mathbf{E}_{0i}) S_i^z \quad (7)$$

for the lowering operator and the Hermitian conjugated equation for the rising operator, with  $k_0 \equiv \omega_0/c$ . The equation for the population-difference operator is

$$\frac{dS_i^z}{dt} = -(k_0 \mathbf{d} \cdot \mathbf{A}_i - i\mathbf{d} \cdot \mathbf{E}_{0i}) S_i^+ - (k_0 \mathbf{d}^* \cdot \mathbf{A}_i + i\mathbf{d}^* \cdot \mathbf{E}_{0i}) S_i^- . \quad (8)$$

In deriving an equation for the vector potential, one has to use the commutation relations

$$[E^\alpha(\mathbf{r}, t), A^\beta(\mathbf{r}', t)] = 4\pi ic \delta_{\alpha\beta}(\mathbf{r} - \mathbf{r}') ,$$

where

$$\delta_{\alpha\beta}(\mathbf{r}) \equiv \frac{1}{(2\pi)^3} \int \left( \delta_{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2} \right) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k}$$

is the so-called transverse delta-function<sup>1</sup>. These commutation relations explicitly take into account the Coulomb gauge condition. The transverse delta-function can be presented in other forms, as

$$\delta_{\alpha\beta}(\mathbf{r}) = \delta_{\alpha\beta} \delta(\mathbf{r}) + \frac{\partial^2}{\partial r^\alpha \partial r^\beta} \int \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{(2\pi)^3 k^2} d\mathbf{k} ,$$

or as

$$\delta_{\alpha\beta}(\mathbf{r}) = \frac{2}{3} \delta_{\alpha\beta} \delta(\mathbf{r}) - \frac{\delta_{\alpha\beta} - 3n_\alpha n_\beta}{4\pi r^3} ,$$

where  $n_\alpha \equiv r^\alpha/r$ . As a result, for the vector potential one has

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{A} = - \frac{4\pi}{c} \mathbf{j} , \quad (9)$$

with the density of current

$$j^\alpha(\mathbf{r}, t) = \sum_\beta \left[ \sum_{i=1}^N \delta_{\alpha\beta}(\mathbf{r} - \mathbf{r}'_i) J_i^\beta(t) + \int \delta_{\alpha\beta}(\mathbf{r} - \mathbf{r}') j_m^\beta(\mathbf{r}', t) d\mathbf{r}' \right] . \quad (10)$$

The solution to Eq. (9) reads

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_{vac}(\mathbf{r}, t) + \frac{1}{c} \int \mathbf{j} \left( \mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c} \right) \frac{d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} , \quad (11)$$

with  $\mathbf{A}_{vac}$  being the solution of the uniform equation related to Eq. (9). Note that the dipolar part of the transverse delta-function, being averaged over space, yields zero, that is

$$\int \delta_{\alpha\beta}(\mathbf{r}) d\mathbf{r} = \frac{2}{3} \delta_{\alpha\beta} .$$

This anisotropic dipolar part, for simplicity, can be omitted. Then the vector potential (11) takes the form

$$\mathbf{A} = \mathbf{A}_{vac} + \mathbf{A}_{rad} + \mathbf{A}_{mat} , \quad (12)$$

in which the term

$$\mathbf{A}_{rad}(\mathbf{r}_i, t) = \sum_{j(\neq i)}^N \frac{2}{3cr_{ij}} \mathbf{J}_j \left( t - \frac{r_{ij}}{c} \right)$$

is due to radiating atoms and the potential

$$\mathbf{A}_{mat}(\mathbf{r}, t) = \frac{2}{3c} \int \mathbf{j}_m \left( \mathbf{r}', t - \frac{1}{c} |\mathbf{r} - \mathbf{r}'| \right) \frac{d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

is caused by matter currents. Here  $r_{ij} \equiv |\mathbf{r}_{ij}|$ ,  $\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$ . The summation over  $j = 1, 2, \dots, N$  does not include the term with  $j = i$  corresponding to self-action, which will be taken into account by incorporating the level and line widths in the evolution equations.

It is important to notice that all processes are defined for  $t \geq 0$ , while in the expressions above there appears the dependence of operators on the difference  $t - t'$ , which may be negative. Therefore it is necessary to complete the definition of quasispin operators by adding the retardation condition

$$S_j^-(t) = 0 \quad (t < 0) . \quad (13)$$

The dependence of  $S_j^-(t - t')$  on the retarded time can also be simplified by taking into account that the interaction of radiation with atoms is to be essentially weaker than intra-atomic interactions. In the other case, the very notion of resonant atoms, having a well-defined transition frequency, would not have sense. Then, Eq. (7) shows that  $S_j^-(t) \sim \exp(-i\omega_0 t)$ . Combining this with the retardation condition (13) gives

$$S_j^-(t - t') = \Theta(t - t') S_j^-(t) \exp(i\omega_0 t') . \quad (14)$$

Hence for the typical dependence on the retarded variables, one has

$$S_j^-\left(t - \frac{r_{ij}}{c}\right) = \Theta(ct - r_{ij}) S_j^-(t) \exp(ik_0 r_{ij}) . \quad (15)$$

Such a simplification, combining the Born approximation with the retardation condition (13), can be called the *Retarded Born Approximation*.

It is worth mentioning that it is sufficient to understand all operator equations in the weak sense, as the equations for appropriate matrix elements or averages.

### 3. Separation of Local Fields

The influence of vacuum and matter on an atom enters the evolution equations through the expression

$$\xi(\mathbf{r}, t) \equiv 2k_0 \mathbf{d} \cdot (\mathbf{A}_{vac} + \mathbf{A}_{mat}) . \quad (16)$$

Actually, electromagnetic vacuum and matter form an effective vacuum acting on an atom by means of the local field (16). Thus, one may say that there are two types of operator variables, the quasispin variables and the local field (16). An operator  $\hat{F}$ , being a function of these variables, can, for brevity, be written as  $\hat{F}(S, \xi)$ , with  $S$  denoting the quasispin set  $\{S_i^\alpha\}$  and  $\xi$ , the set  $\{\xi(\mathbf{r}, t)\}$  of local fields at different points  $\mathbf{r}$ . According to the existence of two types of variables, one may define two kinds of statistical averages over the related variables, either quasispin or local-field ones. Such partial averages are defined by means of the corresponding restricted traces, either

$$\langle \hat{F} \rangle \equiv \text{Tr}_S \hat{\rho} \hat{F}(S, \xi) , \quad (17)$$

with  $\hat{\rho}$  being a statistical operator, or

$$\ll \hat{F} \gg \equiv \text{Tr}_\xi \hat{\rho} \hat{F}(S, \xi) . \quad (18)$$

When averaging over quasispin variables, one may assume the validity of the mean-field decoupling

$$\langle S_i^\alpha S_j^\beta \rangle = \langle S_i^\alpha \rangle \langle S_j^\beta \rangle \quad (i \neq j) , \quad (19)$$

since atoms interact with each other through effective long-range forces. This decoupling is not equivalent to the semiclassical approximation as the local-field variables have not been involved. Then the atomic average  $\langle S_i^\alpha \rangle$  is, actually, an operator function of local fields. The latter describe local quantum fluctuations.

For what follows, it is convenient to use the notation  $S_i^\alpha(t) \equiv S^\alpha(\mathbf{r}_i, t)$ . Also, the summation over atoms can be replaced by the integration over the sample, according to the rule

$$\sum_{i=1}^N \Rightarrow \rho \int d\mathbf{r} \quad \left( \rho \equiv \frac{N}{V} \right) . \quad (20)$$

The behaviour of atoms can be described by the following averages. For an atom at the point  $\mathbf{r}$ , one may write a transition function

$$u(\mathbf{r}, t) \equiv 2 \langle S^-(\mathbf{r}, t) \rangle , \quad (21)$$

intensity of coherence

$$w(\mathbf{r}, t) \equiv u^+(\mathbf{r}, t) u(\mathbf{r}, t) , \quad (22)$$

and population difference

$$s(\mathbf{r}, t) \equiv 2 \langle S^z(\mathbf{r}, t) \rangle . \quad (23)$$

The effective force acting on an atom is given by the sum

$$f(\mathbf{r}, t) = f_0(\mathbf{r}, t) + f_{rad}(\mathbf{r}, t) + \xi(\mathbf{r}, t) , \quad (24)$$

in which

$$f_0(\mathbf{r}, t) \equiv -2i\mathbf{d} \cdot \mathbf{E}_0(\mathbf{r}, t) \quad (25)$$

is due to a classical external field,

$$f_{rad}(\mathbf{r}, t) \equiv 2k_0 < \mathbf{d} \cdot \mathbf{A}_{rad}(\mathbf{r}, t) > \quad (26)$$

is caused by the radiation of other atoms, and  $\xi(\mathbf{r}, t)$  is the local field (16). The radiation force (26) explicitly writes

$$f_{rad}(\mathbf{r}, t) = -i\gamma_0\rho \int \left[ G(\mathbf{r} - \mathbf{r}', t) u(\mathbf{r}', t) - \mathbf{e}_d^2 G^*(\mathbf{r} - \mathbf{r}', t) u^+(\mathbf{r}', t) \right] d\mathbf{r}' , \quad (27)$$

where the transfer function is

$$G(\mathbf{r}, t) \equiv \Theta(ct - r) \frac{\exp(ik_0 r)}{k_0 r}$$

and the notation

$$\gamma_0 \equiv \frac{2}{3} k_0^3 d_0^2 , \quad r \equiv |\mathbf{r}| , \quad \mathbf{d} \equiv d_0 \mathbf{e}_d , \quad d_0 \equiv |\mathbf{d}|$$

is used. The quantity  $\gamma_0$  is a natural half-width.

In this way, for the functions  $u = u(\mathbf{r}, t)$ ,  $w = w(\mathbf{r}, t)$ , and  $s = s(\mathbf{r}, t)$ , we derive the evolution equations

$$\frac{\partial u}{\partial t} = -(i\omega_0 + \gamma_2)u + fs , \quad (28)$$

$$\frac{\partial w}{\partial t} = -2\gamma_2 w + (u^+ f + f^+ u) s , \quad (29)$$

$$\frac{\partial s}{\partial t} = -\frac{1}{2} (u^+ f + f^+ u) - \gamma_1 (s - \zeta) , \quad (30)$$

in which  $\gamma_1$  is a longitudinal relaxation parameter,  $\gamma_2$  is a transverse attenuation parameter, and  $\zeta \in [-1, 1]$  is a stationary population difference for a single atom.

## 4. Sample of Cylindrical Shape

Let the sample have typical for lasers cylindrical shape. The axis of the cylinder is along the  $z$ -axis, which is distinguished by the propagating field

$$\mathbf{E}_0(\mathbf{r}, t) = \frac{1}{2} \mathbf{E}_1 e^{i(kz - \omega t)} + \frac{1}{2} \mathbf{E}_1^* e^{-i(kz - \omega t)} , \quad (31)$$

where the frequency  $\omega = kc$  is close to the transition frequency  $\omega_0$ , so that the resonance condition

$$\frac{|\Delta|}{\omega_0} \ll 1 \quad (\Delta \equiv \omega - \omega_0) \quad (32)$$

holds. The wavelength  $\lambda = 2\pi c/\omega$  is small as compared to the radius,  $R$ , and length,  $L$ , of the cylinder,

$$\frac{\lambda}{R} \ll 1, \quad \frac{\lambda}{L} \ll 1. \quad (33)$$

Assuming the absence of sharp transverse structures, one may employ the single-mode approximation

$$u(\mathbf{r}, t) = u(t)e^{ikz}, \quad w(\mathbf{r}, t) = w(t), \quad s(\mathbf{r}, t) = s(t). \quad (34)$$

The absence of sharp transverse nonuniformity implies that there exists the main propagating mode selected by the field (31). Expressions (34) are to be substituted in Eqs. (28) to (30). Equation (28) is multiplied by  $e^{-ikz}$  and then all equations are averaged over space. The following notation will be used for an effective force

$$f_1(t) \equiv -i\mathbf{d} \cdot \mathbf{E}_1 e^{-i\omega t} + \xi(t), \quad (35)$$

in which

$$\xi(t) \equiv \frac{1}{V} \int \xi(\mathbf{r}, t) e^{-ikz} d\mathbf{r}. \quad (36)$$

Let us also introduce the coupling functions

$$\alpha(t) \equiv \gamma_0 \rho \int \Theta(ct - r) \frac{\sin(k_0 r - kz)}{k_0 r} d\mathbf{r}, \quad \beta(t) \equiv \gamma_0 \rho \int \Theta(ct - r) \frac{\cos(k_0 r - kz)}{k_0 r} d\mathbf{r}. \quad (37)$$

Finally, Eqs. (28) to (30) are transformed into the ordinary differential equations

$$\frac{du}{dt} = -[i(\omega_0 + \beta s) + \gamma_2 - \alpha s]u + f_1 s, \quad (38)$$

$$\frac{dw}{dt} = -2(\gamma_2 - \alpha s)w + (u^+ f_1 + f_1^+ u) s, \quad (39)$$

$$\frac{ds}{dt} = -\alpha w - \frac{1}{2}(u^+ f_1 + f_1^+ u) - \gamma_1(s - \zeta). \quad (40)$$

Although, one should remember that these are, actually, operator equations with respect to the quantum variable  $\xi(t)$ .

## 5. Method of Stochastic Averaging

To simplify further the evolution equations (38) to (40), one can take into consideration the existence of several small parameters, such as

$$\frac{\gamma_0}{\omega_0} \ll 1, \quad \frac{\gamma_1}{\omega_0} \ll 1, \quad \frac{\gamma_2}{\omega_0} \ll 1. \quad (41)$$

The amplitude of the external field (31) is assumed to be small, so that

$$\frac{|\nu_1|}{\omega_0} \ll 1, \quad \nu_1 \equiv \mathbf{d} \cdot \mathbf{E}_1. \quad (42)$$

Also, the local quantum field  $\xi$  is treated as weak, in the sense that its first and second moments are proportional to values much smaller than  $\omega_0$ . Then the multiscale averaging technique can be generalized<sup>3-7</sup> to stochastic and operator equations, as Eqs. (38) to (40). The occurrence of the above small parameters shows that the functions  $w(t)$  and  $s(t)$  are temporal quasi-invariants with respect to the fast function  $u(t)$ .

Equation (38) for the fast function, with  $w$  and  $s$  being quasi-invariants, can be solved. To this end, let us introduce the *collective width* and *collective frequency* by the corresponding expressions

$$\Gamma \equiv \gamma_2 - \alpha s, \quad \Omega \equiv \omega_0 + \beta s, \quad (43)$$

and also, let us define the *dynamical detuning*

$$\delta \equiv \omega - \Omega = \Delta - \beta s. \quad (44)$$

The solution of Eq. (38) reads

$$u = \left( u_0 - \frac{\nu_1 s}{\delta + i\Gamma} \right) e^{-(i\Omega + \Gamma)t} + \frac{\nu_1 s}{\delta + i\Gamma} e^{-i\omega t} + s \int_0^t \xi(t') e^{-(i\Omega + \Gamma)(t-t')} dt'. \quad (45)$$

To simplify the following formulas, it is convenient to choose the phase of  $\mathbf{E}_1$  so that to eliminate the dependence on the transverse initial value  $u_0$ . For this purpose, the phase of  $\mathbf{E}_1$  is taken so that  $u_0^* \mathbf{d} \cdot \mathbf{E}_1$  be real, which writes

$$u_0^* \nu_1 = u_0 \nu_1^*. \quad (46)$$

The solution (45) is to be substituted into Eqs. (39) and (40), whose right-hand sides are to be averaged according to the prescription

$$\ll \overline{F} \gg = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \ll F(\xi, t) \gg dt,$$

where the integration over time does not touch quasi-invariants. The quantum field  $\xi(t)$  is centered so that

$$\ll \xi(t) \gg = 0. \quad (47)$$

To make the resulting expressions less cumbersome, let us consider the case of small detuning  $|\delta| < |\Gamma|$ , when  $\delta$  can be omitted in the phase dependence, though should be kept in denominators to avoid spurious poles. In the process of the averaging, one obtains the *effective attenuation*

$$\tilde{\Gamma} \equiv \frac{|\nu_1|^2 \Gamma}{\delta^2 + \Gamma^2} (1 - e^{-\Gamma t}) + \Gamma_3, \quad (48)$$

where the first term is due to the classical external field (31) and the *quantum attenuation*

$$\Gamma_3 \equiv \text{Re} \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \int_0^t \ll \xi^+(t) \xi(t') \gg e^{-(i\Omega + \Gamma)(t-t')} dt' \quad (49)$$

appears because of the action of the quantum field  $\xi(t)$ .



Equations (39) and (40) reduce to the evolution equations

$$\frac{dw}{dt} = -2(\gamma_2 - \alpha s)w + 2\tilde{\Gamma}s^2, \quad \frac{ds}{dt} = -\alpha w - \tilde{\Gamma}s - \gamma_1(s - \zeta), \quad (50)$$

describing the coherent guiding centers. In order to make these equations complete, it is necessary to define the quantum correlation function  $\ll \xi^+(t)\xi(t') \gg$  entering the quantum attenuation (49).

## 6. Examples of Quantum Attenuation

To make it clear how the quantum attenuation (49) can be calculated, let us give some examples of defining the quantum variable  $\xi(t)$ .

The first simple case could be by considering the variable  $\xi(t)$  as random, associated to infrared noise characterized by

$$\ll \xi^+(t)\xi(t') \gg = \gamma_3^2. \quad (51)$$

Then Eq. (49) yields

$$\Gamma_3 = \frac{\gamma_3^2 \Gamma}{\Omega^2 + \Gamma^2}. \quad (52)$$

The opposite case would be to treat  $\xi(t)$  as a stochastic variable representing white noise, with

$$\ll \xi^+(t)\xi(t') \gg = 2\Gamma_3\delta(t - t'), \quad (53)$$

which results in the identity  $\Gamma_3 = \Gamma_3$ .

A more elaborate modelling of the effective quantum fluctuations is by a system of oscillators, yielding

$$\xi(t) = \sum_q \gamma(\omega_q) (b_q e^{-i\omega_q t} + b_q^\dagger e^{i\omega_q t}), \quad (54)$$

where  $\omega_q = \omega_{-q} > 0$ . For the Bose operators  $b_q$  and  $b_q^\dagger$ , statistical averaging gives

$$\ll b_q \gg = 0, \quad \ll b_q^\dagger b_p \gg = n_q \delta_{qp}, \quad \ll b_q b_p^\dagger \gg = (1 + n_q) \delta_{qp}, \quad (55)$$

with  $n_q$  being a momentum distribution function. The averages  $\ll b_q b_q \gg$  and  $\ll b_q^\dagger b_q^\dagger \gg$  in the case of normal effective vacuum are zero, while for a squeezed vacuum

$$\ll b_q^\dagger b_p^\dagger \gg = m_q \Delta(\omega_q + \omega_p - 2\omega_s), \quad (56)$$

where the function  $m_q$  is defined by the particular properties of a squeezed vacuum, and  $\Delta(\omega)$  is the discrete delta-function

$$\Delta(\omega) \equiv \begin{cases} 1, & \omega = 0 \\ 0, & \omega \neq 0. \end{cases}$$

The spectral function  $\gamma(\omega)$  in Eq. (54) is assumed to be symmetric with respect to the central line, so that

$$\gamma(\omega_s + \omega_q) = \gamma(\omega_s - \omega_q) .$$

Such an effective squeezed vacuum can be realized if atoms are inserted into a medium with squeezed collective excitations interacting with electromagnetic field. These excitations could be squeezed optical phonons or magnons. The interaction of the latter with photons results in the formation of squeezed polaritons.

For the quantum field (54), the correlation function is

$$\begin{aligned} \ll \xi^+(t)\xi(t') \gg = \sum_q \gamma^2(\omega_q) & \left[ n_q e^{i\omega_q(t-t')} + (1 + n_q) e^{-i\omega_q(t-t')} + \right. \\ & \left. + m_q e^{i\omega_q t + i(2\omega_s - \omega_q)t'} + m_q^* e^{-i\omega_q t - i(2\omega_s - \omega_q)t'} \right] . \end{aligned} \quad (57)$$

Calculating the quantum attenuation (49), we keep in mind that  $n_q \equiv n(\omega_q)$  is real, while  $m_q \equiv m(\omega_q)$  is, in general, complex,

$$m(\omega) = |m(\omega)| e^{i\varphi_s} .$$

The final formulas will be simplified by remembering that  $|\delta| \equiv |\omega - \Omega| \ll \omega$ . Then substituting the correlation function (57) into Eq. (49), one finds

$$\Gamma_3 = \Gamma \sum_q \gamma^2(\omega_q) \left[ \frac{n_q}{(\Omega - \omega_q)^2 + \Gamma^2} + \frac{1 + n_q}{(\Omega + \omega_q)^2 + \Gamma^2} \right] - n(\omega) \frac{\gamma^2(\omega)\Gamma}{\delta^2 + \Gamma^2} e^{-\Gamma t} + \Gamma_s , \quad (58)$$

with the last term being due to the squeezing,

$$\Gamma_s = -|m(\omega)| \gamma^2(\omega) \frac{\Gamma \cos \varphi_s + 2\omega_s \sin \varphi_s}{4\omega_s^2 + \Gamma^2} e^{-\Gamma t} . \quad (59)$$

The quantity (59) may be named the *squeezing attenuation*.

Separating out of the quantum attenuation (58) its resonant part, one has

$$\Gamma_{res} = n(\omega) \frac{\gamma^2(\omega)\Gamma}{\delta^2 + \Gamma^2} (1 - e^{-\Gamma t}) . \quad (60)$$

As is seen, the *resonant attenuation* (60) is zero at  $t = 0$ . This means that, at the initial time, the main role in the quantum attenuation is played by its nonresonant parts, including the squeezing attenuation.

To complete this section, it is useful to present explicit examples of  $n(\omega)$  and  $|m(\omega)|$  in the case of a squeezed effective vacuum. The latter can be generated by a parametric oscillator<sup>1</sup>, with a squeezing field proportional to  $\varepsilon \cos(2\omega_s t + \varphi_s)$ . One employs the notation  $\mu \equiv \gamma - \varepsilon$  and

$\nu \equiv \gamma + \varepsilon$ , where  $\gamma$  is a cavity damping rate. Then, for a non-degenerate parametric oscillator, one has

$$n(\omega) = \frac{\nu^2 - \mu^2}{8} \left[ \frac{1}{(\Delta_s + \kappa)^2 + \mu^2} + \frac{1}{(\Delta_s - \kappa)^2 + \mu^2} - \frac{1}{(\Delta_s + \kappa)^2 + \nu^2} - \frac{1}{(\Delta_s - \kappa)^2 + \nu^2} \right],$$

$$|m(\omega)| = \frac{\nu^2 - \mu^2}{8} \left[ \frac{1}{(\Delta_s + \kappa)^2 + \mu^2} + \frac{1}{(\Delta_s - \kappa)^2 + \mu^2} + \frac{1}{(\Delta_s + \kappa)^2 + \nu^2} + \frac{1}{(\Delta_s - \kappa)^2 + \nu^2} \right],$$

where  $\Delta_s \equiv \omega - \omega_s$ . The parameter  $\kappa$  characterizes a two-mode squeezed field, representing the displacement from the central frequency of squeezing, where the two-mode squeezed vacuum is maximally squeezed. If the parametric oscillator is weakly nondegenerate, with  $|\kappa| \ll |\Delta_s|$ , or degenerate, then

$$n(\omega) = \frac{\nu^2 - \mu^2}{4} \left( \frac{1}{\Delta_s^2 + \mu^2} - \frac{1}{\Delta_s^2 + \nu^2} \right), \quad |m(\omega)| = \frac{\nu^2 - \mu^2}{4} \left( \frac{1}{\Delta_s^2 + \mu^2} + \frac{1}{\Delta_s^2 + \nu^2} \right).$$

By means of different parametric oscillators, it is possible to generate squeezed fields with various characteristics.

## 7. Conclusion

The method, presented in this report, makes it possible to combine the clarity of the semi-classical approximation with taking account of quantum effects. The latter are responsible for the appearance of a specific quantum attenuation. The vacuum electromagnetic field interacting with matter forms an effective quantum vacuum. Preparing different kinds of such vacua, one can regulate the value of the quantum attenuation. The latter, in its turn, can essentially influence the features of the following coherent processes.

Notice that the squeezing attenuation (59) can change sign for the varying phase  $\varphi_s$ . Thus,  $\Gamma_s < 0$  for  $\varphi_s = 0, \pi/2$ .

For preparing different types of effective vacua, one may incorporate resonant atoms into different media. There exists a rich variety of materials possessing various properties, which could be used for creating effective vacua with divers specific features. Just as a few cases of materials with interesting electromagnetic properties, one may mention biopolymers with photoexcited polar vibrations<sup>8</sup>, photonic crystals with two-photon transitions<sup>9</sup>, and plausible inverted superconductors<sup>10</sup>.

The feasibility of influencing coherent processes by quantum effects opens a novel way of regulating the characteristics of collective atomic radiation.

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